CBSE Board
Class XI Mathematics
Sample Paper – 9

Time: 3 hrs **Total Marks: 100**

General Instructions:

1. All questions are compulsory.
2. The question paper consist of 29 questions.
3. Questions 1 – 4 in Section A are very short answer type questions carrying 1 mark each.
4. Questions 5 – 12 in Section B are short-answer type questions carrying 2 mark each.
5. Questions 13 – 23 in Section C are long-answer I type questions carrying 4 mark each.
6. Questions 24 – 29 in Section D are long-answer type II questions carrying 6 mark each.

SECTION – A

1. Find $\lim\limits_{x\to 0}\frac{\sin ax}{bx}$

2. Is the given sentence statement? Justify. "There are 35 days in a month."

3. Write in the form of a + bi : $\frac{1}{i-1}$

OR

Find modulus of 2i.

4. If variance of 20 observations is 5. If each observation is multiplied by 2, then find variance of the new observations.

SECTION – B

5. Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8} verify that A × C is a subset of B × D.

6. Let f be defined by f(x) = x – 4 and g be defined by

$$g(x)=\frac{x^2-16}{x+4} \qquad x\neq -4$$
$$=\lambda \qquad x=-4$$

Find λ such that f(x) = g(x) for all x.

OR

Find domain and range of the function $f(x)=\frac{x^2-9}{x-3}$

7. Assuming that a person of normal sight can read print at such a distance that the letters subtend an angle of 5' at this eye, find the height of the letters that he can read at a distance of 12 m.

OR

If the arcs of the same length in two circles subtend angles of 60° and 75° at their centres. Find the ratio of their radii.

8. If n(U) = 600, n(A) = 460, n(B) = 390 and n(A ∩ B) = 325 then find n(A ∪ B) and n(A ∪ B)'

9. Prove that $\sin(\theta + 30^\circ) = \cos\theta + \sin(\theta - 30^\circ)$

OR

Prove that $\frac{\sin 7A - \sin 5A}{\cos 5A + \cos 7A} = \tan A$

10. Find compound statements of the "It is raining and it is cold."

11. If $f(x) = x^2$ find $\frac{f(1.1)-f(1)}{1.1-1}$

12. Find the equation of line joining the points (-1, 3) and (4, -2).

SECTION – C

13. Prove that $\frac{\cos^2 33^\circ - \cos^2 57^\circ}{\sin^2 \frac{21^\circ}{2} - \sin^2 \frac{69^\circ}{2}} = -\sqrt{2}$

14. If f is a real function defined by $f(x)=\frac{x-1}{x+1}$ then prove that $f(2x)=\frac{3f(x)+1}{f(x)+3}$

15. Let f : R → R be given by $f(x) = x^2 + 3$. Find

i. $\{x : f(x) = 28\}$

ii. The pre-image of 39 and 2 under f.

16. A man accepts a position with an initial salary of Rs. 5200 per week. It is understood that he will receive an automatic increase of Rs. 320 in the very next and each month.

i. find his salary for the tenth month

ii. his total earning during the first year.

17. If $(x + yi)^3 = u + vi$ prove that $\frac{u}{x}+\frac{v}{y}=4\left(x^2+y^2\right)$

18. Two cards are drawn from a pack of cards. What is the probability that either both are red or both are kings?

19. Determine the number n in a geometric progression $\{a_n\}$, if $a_1 = 3$, $a_n = 96$ and $S_n = 189$.

20. Find n, if ${}^{2n}C_1$, ${}^{2n}C_2$ and ${}^{2n}C_3$ are in A. P.

OR

Prove that the product of 2n consecutive negative integers is divisible by (2n)!.

21. Find the equation of the straight line through the origin making angle of 60° with the straight line $x + \sqrt{3}\,y + 3\sqrt{3} = 0$

OR

Find the equations of the lines, which cut off intercepts on the axes whose sum and product are 1 and -6 respectively.

22. Differentiate $x^{-3/2}$ with respect to x using first principle.

OR

Differentiate $\frac{x+2}{x^2+3}$ and find the value of derivative at x = 0.

23. Find the equation of hyperbola whose foci are (8, 3) and (0, 3) and e = 4/3

SECTION – D

24. Prove that $\cot\theta \cot 2\theta + \cot 2\theta \cot 3\theta + 2 = \cot\theta(\cot\theta - \cot 3\theta)$

OR

Prove that $5\cos\theta + 3\cos\left(\theta+\frac{\pi}{3}\right) + 3$ lies between -4 and 10.

25. Find the mean and variance of the following data

Classes	0 - 30	30 - 60	60 – 90	90 - 120	120 - 150	150 - 180	180 - 210
Frequency	2	3	5	10	3	5	2

26. If Find $\sin\frac{x}{2}, \cos\frac{x}{2}$ and $\tan\frac{x}{2}$ where $\tan x = -\frac{4}{3}$, x is in quadrant II

27. Plot the given linear inequations and shade the region which is common to the solution of all inequations $x \geq 0$, $y \geq 0$, $5x + 3y \leq 500$; $x \leq 70$ and $y \leq 125$.

OR

How many litres of water will have to be added to 1125 litres of a 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

28. Using principle of mathematical induction prove that $5^n - 5$ is divisible by 4 for all $n \in N$. Hence, prove that $2 \times 7^n + 3 \times 5^n - 5$ is divisible by 24 for all $n \in N$.

29. If a, b, and c are in A.P.; b, c, and d are in G.P. and $\frac{1}{c}, \frac{1}{d}$, and $\frac{1}{e}$ are in A.P., prove that a, c, and e are in G.P.

OR

Show that:

$$\frac{1\times 2^2 + 2\times 3^2 + .. + n\times(n+1)^2}{1^2\times 2 + 2^2\times 3 + .. + n^2\times(n+1)} = \frac{3n+5}{3n+1}$$

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Class XI Mathematics
Sample Paper – 9 Solution

SECTION – A

1.

$$\lim_{x\to 0}\frac{\sin ax}{bx}$$
$$=\lim_{x\to 0}\frac{\sin ax}{ax}\times\frac{a}{b}$$
$$=\frac{a}{b}$$

2. The sentence is false because a month can't have more than 31 days. Hence it is a statement.

3.

$$\frac{1}{i-1}$$
$$=\frac{1}{-1+i}$$
$$=\frac{1}{-1+i}\times\frac{-1-i}{-1-i}$$
$$=\frac{-1-i}{1-i^2}$$
$$=\frac{-1-i}{1-(-1)}$$
$$=\frac{-1-i}{2}$$
$$=\frac{-1}{2}-\frac{i}{2}$$

a = -1/2 and b = -1/2

OR

z = 2i

Comparing with a + bi we get a = 0 and b = 2

$$|z|=\sqrt{0+2^2}=2$$

4. According to the question,

Variance of 20 observations is 5.

New variance = $2^2 \times 5 = 20$

SECTION – B

5. A × C = {(1, 5),(1, 6), (2, 5), (2, 6)}....(i)

B × D = {(1, 5),(1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}.......(ii)

A × C is a subset of B × D all the elements of A × C are contained in B × D.

6. According to the question,

f(x) = g(x)

f(-4) = g(-4)

$-4 - 4 = \lambda$

$\lambda = -8$

OR

$$f(x) = \frac{x^2 - 9}{x - 3}$$

f(x) is not defined for x – 3 = 0 i. e. x = 3. Therefore, domain (f) = r – {3}

Let f(x) = y then

$$\frac{x^2 - 9}{x - 3} = y$$

x + 3 = y

It follows from the above relation that y takes all real values except 6 when x takes values in the set R – {3}. Therefore, Range (f) = R – {6}

7. Let h be the required height in metres. Here h can be considered as the arc of a circle of radius 12 m, which subtends an angle of 5’ at its centre.

$$\theta = 5' = \left(\frac{5}{60}\right)^{\circ} = \left(\frac{1}{12} \times \frac{\pi}{180}\right)^{c} \text{ and } r = 12 \text{ m}$$

$$\theta = \frac{arc}{r} = \frac{\pi}{12 \times 180} = \frac{h}{12}$$

$$h = \frac{\pi}{180} = 1.7 \text{ m}$$

OR

Let r_1 and r_2 be the radii of the given circles and let their arcs of same length s subtend angles of 60° and 75° at their centres.

$$60° = 60° \times \frac{\pi}{180} = \frac{\pi}{3}$$

$$75° = 75° \times \frac{\pi}{180} = \frac{5\pi}{12}$$

$$s = r_1\theta_1 = r_2\theta_2$$

$$\frac{\pi}{3} r_1 = \frac{5\pi}{12} r_2$$

$$\frac{r_1}{r_2} = \frac{5}{12} \times 3$$

$$\frac{r_1}{r_2} = \frac{5}{4}$$

8. n(U) = 600, n(A) = 460, n(B) = 390 and n(A ∩ B) = 325

n(A ∪ B) = n(A) + n(B) – n(A ∩ B)

= 460 + 390 – 325

= 525

n(A ∪ B)' = n(U) – n(A ∪ B)

= 600 – 525

= 75

9. $\sin(\theta + 30°) = \sin\theta \cos 30° + \cos 30° \sin\theta$

$= (\sin\theta \cos 30° - \cos\theta \sin 30°) + 2\cos\theta \sin 30°$

$= \sin(\theta - 30°) + 2\cos\theta \times \frac{1}{2}$

$= \cos\theta + \sin(\theta - 30°)$

$\therefore \sin(\theta + 30°) = \cos\theta + \sin(\theta - 30°)$

OR

$$\frac{\sin 7A - \sin 5A}{\cos 5A + \cos 7A}$$

$$= \frac{2\cos\frac{7A+5A}{2}\sin\frac{7A-5A}{2}}{2\cos\frac{7A+5A}{2}\cos\frac{7A-5A}{2}}$$

$$= \frac{\cos 6A \sin A}{\cos 6A \cos A}$$

$$= \frac{\sin A}{\cos A}$$

$$= \tan A$$

10. The compound statements are

p : It is raining

q : It is cold.

11. $f(x) = x^2$

$$\frac{f(1.1)-f(1)}{1.1-1}$$

$$=\frac{1.1^2-1^2}{1.1-1} \quad s$$

$$=\frac{1.21-1}{0.1}$$

$$=2.1$$

12. $(x_1, y_1) = (-1, 3)$ and $(x_2, y_2) = (4, -2)$

The equation of line is $y - 3 = \frac{3-(-2)}{-1-4}(x + 1)$

$y - 3 = -x - 1$

$x + y - 2 = 0$

SECTION – C

13.

$$\frac{\cos^2 33° - \cos^2 57°}{\sin^2\frac{21°}{2} - \sin^2\frac{69°}{2}}$$

$$=\frac{1-\sin^2 33° - \left(1-\sin^2 57°\right)}{\sin^2\frac{21°}{2} - \sin^2\frac{69°}{2}}$$

$$=\frac{\sin^2 57° - \sin^2 33°}{\sin^2\frac{21°}{2} - \sin^2\frac{69°}{2}}$$

$$=\frac{\sin(57°+33°)\sin(57°-33°)}{\sin\left(\frac{21°}{2}+\frac{69°}{2}\right)\sin\left(\frac{21°}{2}+\frac{69°}{2}\right)}$$

$$=\frac{\sin 90° \sin 24°}{\sin 45° \sin(-24°)}$$

$$=\frac{\sin 24°}{\frac{-1}{\sqrt{2}}\sin 24°} = -\sqrt{2}$$

14. $f(x)=\dfrac{x-1}{x+1}$

$$\frac{f(x)}{1}=\frac{x-1}{x+1}$$

$$\frac{f(x)+1}{f(x)-1}=\frac{x-1+x+1}{x-1-x-1}$$

$$\frac{f(x)+1}{f(x)-1}=-x$$

$$\frac{f(x)+1}{1-f(x)}=x$$

$$f(2x)=\frac{2x-1}{2x+1}$$

$$f(2x)=\frac{2\times\frac{f(x)+1}{1-f(x)}-1}{2\times\frac{f(x)+1}{1-f(x)}+1}$$

$$f(2x)=\frac{2\times f(x)+2-1+f(x)}{2\times f(x)+2+1-f(x)}$$

$$f(2x)=\frac{3f(x)+1}{f(x)+3}$$

15. i. $f(x) = x^2 + 3$

$f(x) = 28$

$x^2 + 3 = 28$

$x^2 = 25$

$x = \pm 5$

$\{x : f(x) = 28\} = \{5, -5\}$

ii. Let x be the pre-image of 39. Then,

$f(x) = 39$

$x^2 + 3 = 39$

$x^2 = 36$

$x = \pm 6$

So, pre-image of 39 are -6 and 6.

Let x be the pre-image of 2. Then,

$f(x) = 2$

$x^2 + 3 = 2$

$x^2 = -1$

We find that no real value of x satisfies the equation. Therefore, 2 does not have any pre-image under f.

16. Salary in the first week = 5200 × 4 = Rs. 20800

Increase = Rs. 320 per week

a = 20800 and d = 320

i. Salary in the 10^{th} month = t_{10}

$= a + 9d$

$= 20800 + 9 \times 320$

= Rs. 23680

ii. Total earning during the first year.

$= \frac{12}{2}\left[2\times20800+(12-1)\times320\right]$

= 6 (41600 + 3520)

= Rs. 270720

17. $(x + yi)^3 = u + vi$

$x^3 + 3x^2yi + 3xy^2i^2 + y^3i^3 = u + vi$

$x^3 + 3x^2yi - 3xy^2 - y^3i = u + vi$

$x^3 - 3xy^2 + (3x^2y - y^3)i = u + vi$

$x^3 - 3xy^2 = u$ and $3x^2y - y^3 = v$

$x(x^2 - 3y^2) = u$ and $y(3x^2 - y^2) = v$

$$\frac{u}{x}+\frac{v}{y}=x^2-3y^2+3x^2-y^2$$

$$\frac{u}{x}+\frac{v}{y}=4x^2-4y^2$$

$$\frac{u}{x}+\frac{v}{y}=4\left(x^2-y^2\right)$$

18. Two cards out of 52 cards. Can be drawn in ${}^{52}C_2$ ways.

$n(S) = {}^{52}C_2$

Let A be the event that two cards are drawn are red cards and B be the event that two cards are drawn are kings.

$n(A) = {}^{26}C_2$ and $n(B) = {}^{4}C_2$

Required probability = $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A) = \frac{n(A)}{n(S)}=\frac{{}^{26}C_2}{{}^{52}C_2}$ and $P(B) = \frac{n(B)}{n(S)}=\frac{{}^{4}C_2}{{}^{52}C_2}$

Also there are two cards which are both red and kings.

$P(A \cap B)$ = Probability of getting two red kings = $\frac{{}^{2}C_2}{{}^{52}C_2}$

Required probability = $\frac{{}^{26}C_2}{{}^{52}C_2}+\frac{{}^{4}C_2}{{}^{52}C_2}-\frac{{}^{2}C_2}{{}^{52}C_2}=\frac{325}{1326}+\frac{1}{221}-\frac{1}{1326}=\frac{55}{221}$

19. Let r be the common ratio.

$a_n = a_1 r^{n-1} = 3 r^{n-1}$

$3 r^{n-1} = 96$

$r^{n-1} = 32$......(i)

$S_n = \frac{a_1(r^n - 1)}{r-1} = \frac{3(r^n - 1)}{r-1} = 189$

$\frac{r^n - 1}{r-1} = 63$

$r^n - 1 = 63r - 63$

$r^n = 63r - 62$

$r^{n-1} \cdot r = 63r - 62$

$32r = 63r - 62$ from (i)

$31r = 62$

$r = 2$

$2^{n-1} = 32 = 2^5$

$n - 1 = 5$

$n = 6$

20. Since, ${}^{2n}C_1$, ${}^{2n}C_2$ and ${}^{2n}C_3$ are in A. P.

$2\,{}^{2n}C_2 = {}^{2n}C_1 + {}^{2n}C_3$

$2\frac{2n(2n-1)}{2\times1} = 2n + \frac{2n(2n-1)(2n-2)}{3\times2\times1}$

$2n(2n-1) = 2n + \frac{1}{3}n(2n-1)(2n-2)$

$6n(2n-1) = 6n + n(2n-1)(2n-2)$

$6(2n-1) = 6 + (2n-1)(2n-2)$ $\because n \neq 0$

$12n - 6 = 6 + 4n^2 - 2n - 4n + 2$

$4n^2 - 18n + 14 = 0$

$2n^2 - 9n + 7 = 0$

$n = \frac{9 \pm \sqrt{81-56}}{4} = \frac{9 \pm 5}{4} = \frac{7}{2}, 1$

But $n \neq 1$ as ${}^2C_3$ not possible hence, $n = 7/2$

OR

Let –r, -r – 1, -r – 2,.....(-r – 2n + 1) be 2n consecutive negative integers.

Then their product = (-r) (-r – 1)(-r – 1).........(-r – 2n + 1)

$= (-1)^{2n} r\,(r + 1)(r + 2)......(r + 2n - 1)$

$$= \frac{(r-1)!\,r(r+1)(r+2)......(r+2n-1)}{(r-1)!}$$

$$= \frac{(r+2n-1)!}{(r-1)!}$$

$$= \frac{(r+2n-1)!(2n)!}{(r-1)!(2n)!}$$

$$= {}^{r+2n-1}C_{2n}\,(2n)!$$

Hence, the product is divisible by (2n)!.

21. The equation of any straight line through (0, 0) is

$y = 0 = m(x - 0)$

$y = mx$........(i)

The given straight line is $x + \sqrt{3}\,y + 3\sqrt{3} = 0$........(ii)

Thus $m_1 = m$ and $m_2 = \dfrac{-\text{coefficient of } x}{\text{coefficient of } y} = \dfrac{-1}{\sqrt{3}}$

By the question, angle between (i) and (ii) is 60°.

$$\left|\frac{m-\left(\frac{-1}{\sqrt{3}}\right)}{1+m\left(\frac{-1}{\sqrt{3}}\right)}\right| = \tan 60°$$

$$\pm\frac{\sqrt{3}m+1}{\sqrt{3}-m} = \sqrt{3}$$

$$\frac{\sqrt{3}m+1}{\sqrt{3}-m} = \sqrt{3}$$

$$\sqrt{3}m+1 = \sqrt{3}\left(\sqrt{3}-m\right)$$

$$\sqrt{3}m+1 = 3-\sqrt{3}m$$

$$2\sqrt{3}m = 2$$

$$m = \frac{1}{\sqrt{3}}$$

$$-\frac{\sqrt{3}m+1}{\sqrt{3}-m} = \sqrt{3}$$

$$-\sqrt{3}m-1 = \sqrt{3}\left(\sqrt{3}-m\right)$$

$$-\sqrt{3}m-1 = 3-\sqrt{3}m$$

Here, m is undefined.

Hence, the equation of the straight line is $y = \frac{1}{\sqrt{3}}x$

OR

Let the intercepts be a and b so that a + b = 1……(i)
ab = -6……(ii)
b = 1 – a…………..(iii)
a(1 – a) = -6
$a^2 - a + 6 = 0$ wrong step
(a – 3)(a + 2) = 0
a = 3 or a = -2
When a = 3 then b = 1 – 3 = -2
When a = -2 then b – 1 – (-2) = 3
The equations of the straight lines are

$$\frac{x}{3}+\frac{y}{-2}=1 \text{ or } \frac{x}{-2}+\frac{y}{3}=1$$

2x – 3y – 6 = 0 or 3x – 2y + 6 = 0

22. $f(x) = x^{-3/2} \therefore f(x + h) = (x + h)^{-3/2}$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^{-\frac{3}{2}}-x^{\frac{-3}{2}}}{h}$$

$$f'(x)=\lim_{h\to 0}\frac{(x+h)^{-\frac{3}{2}}-x^{\frac{-3}{2}}}{x+h-x}$$

Put z = x + h and z → x as h → 0

$$f'(x)=\lim_{z\to x}\frac{z^{-\frac{3}{2}}-x^{\frac{-3}{2}}}{z-x}$$

$$f'(x)=-\frac{3}{2}x^{-\frac{3}{2}-1}$$

$$f'(x)=-\frac{3}{2}x^{-\frac{5}{2}}$$

OR

$$y=\frac{x+2}{x^2+3}$$

$$\frac{dy}{dx}=\frac{\left(x^2+3\right)\frac{d}{dx}(x+2)-(x+2)\frac{d}{dx}\left(x^2+3\right)}{\left(x^2+3\right)^2}$$

$$\frac{dy}{dx}=\frac{x^2+3-(x+2)\times 2x}{\left(x^2+3\right)^2}$$

$$\frac{dy}{dx}=\frac{x^2+3-2x^2-4x}{\left(x^2+3\right)^2}$$

$$\frac{dy}{dx}=\frac{-x^2-4x+3}{\left(x^2+3\right)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=0}=\frac{-0+0+3}{(0+3)^2}=\frac{3}{9}=\frac{1}{3}$$

23. The centre of the hyperbola is the mid-point of the line joining the two foci.

Centre = $\left(\frac{8+0}{2},\frac{3+3}{2}\right)$ i. e. (4, 3)

Since the foci lie on the line y = 3 the equation of the hyperbola is of the form

$\frac{(x-4)^2}{a^2}-\frac{(y-3)^2}{b^2}=1$ where 2 and 2b are the transverse and conjugate axis respectively.

Distance between foci = $\sqrt{(8-0)^2+(3-3)^2}=8=2ae$

$$2a\times\frac{4}{3}=8$$

$$a=3$$

$$b^2=a^2\left(e^2-1\right)=9\left(\frac{16}{9}-1\right)=7$$

The equation of hyperbola is

$$\frac{(x-4)^2}{9}-\frac{(y-3)^2}{7}=1$$

$$7x^2-9y^2-56x+54y-32=0$$

SECTION – D

24. cot θ cot 2θ + cot 2θ cot 3θ + 2

$= \cot\theta\cot 2\theta + 1 + \cot 2\theta\cot 3\theta + 1$

$$= \frac{\cos\theta\cos 2\theta}{\sin\theta\sin 2\theta} + 1 + \frac{\cos 2\theta\cos 3\theta}{\sin 2\theta\sin 3\theta} + 1$$

$$= \frac{\cos\theta\cos 2\theta + \sin\theta\sin 2\theta}{\sin\theta\sin 2\theta} + \frac{\cos 2\theta\cos 3\theta + \sin 2\theta\sin 3\theta}{\sin 2\theta\sin 3\theta}$$

$$= \frac{\cos(2\theta-\theta)}{\sin\theta\sin 2\theta} + \frac{\cos(3\theta-2\theta)}{\sin 2\theta\sin 3\theta}$$

$$= \frac{\cos\theta}{\sin\theta\sin 2\theta} + \frac{\cos\theta}{\sin 2\theta\sin 3\theta}$$

$$= \cos\theta\left(\frac{1}{\sin\theta\sin 2\theta} + \frac{1}{\sin 2\theta\sin 3\theta}\right)$$

$$= \frac{\cos\theta}{\sin\theta}\left(\frac{\sin\theta}{\sin\theta\sin 2\theta} + \frac{\sin\theta}{\sin 2\theta\sin 3\theta}\right)$$

$$= \cot\theta\left(\frac{\sin(2\theta-\theta)}{\sin\theta\sin 2\theta} + \frac{\sin(3\theta-2\theta)}{\sin 2\theta\sin 3\theta}\right)$$

$$= \cot\theta\left(\frac{\sin 2\theta\cos\theta - \cos 2\theta\sin\theta}{\sin\theta\sin 2\theta} + \frac{\sin 3\theta\cos 2\theta - \cos 3\theta\sin 2\theta}{\sin 2\theta\sin 3\theta}\right)$$

$$= \cot\theta(\cot\theta - \cot 2\theta + \cot 2\theta - \cot 3\theta)$$

$= \cot\theta(\cot\theta - \cot 3\theta)$

$\cot\theta\cot 2\theta + \cot 2\theta\cot 3\theta + 2 = \cot\theta(\cot\theta - \cot 3\theta)$

OR

Let $f(\theta) = 5\cos\theta + 3\cos\left(\theta + \frac{\pi}{3}\right) + 3$

$f(\theta) = 5\cos\theta + 3\cos\left(\cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3}\right) + 3$

$= 5\cos\theta + 3\cos\left(\frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta\right) + 3$

$= \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$$-\sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2} \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta \le \sqrt{\left(\frac{13}{2}\right)^2 + \left(\frac{3\sqrt{3}}{2}\right)^2}$$

$$-7 \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta \le 7$$

$$-7+3 \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3 \le 7+3$$

$$-4 \le \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3 \le 10$$

25. Let assumed mean be A = 105

Classes	f_i	x_i	$u_i = \frac{x_i - 105}{30}$	f_iu_i	$f_iu_i^2$
0 – 30	2	15	-3	-6	18
30 – 60	3	45	-2	-6	12
60 – 90	5	75	-1	-5	5
90 – 120	10	105	0	0	0
120 – 150	3	135	1	3	3
150 – 180	5	165	2	10	20
180 - 210	2	195	3	6	18
	$\sum f_i = 30$			$\sum f_iu_i = 2$	$\sum f_iu_i^2 = 76$

Mean $\bar{X} = A + \frac{\sum f_iu_i}{\sum f_i} \times h$

$= 105 + \frac{2}{30} \times 30$

$= 107$

Variance $\sigma^2 = h^2 \times \left[\frac{1}{N}\sum f_iu_i^2 - \left(\frac{1}{N}\sum f_iu_i\right)^2\right]$

$= (30)^2 \left[\frac{1}{30} \times 76 - \left(\frac{2}{30}\right)^2\right]$

$= 900 \left[\frac{76}{30} - \frac{4}{900}\right] = 2276$

26.

$\tan x = -\frac{4}{3}$; $\frac{\pi}{2} \le x \le \pi$

$$\tan x = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \qquad \left(\because \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$\Rightarrow -\frac{4}{3} = \frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}} \Rightarrow 4\left(1-\tan^2\frac{x}{2}\right) = -6\tan\frac{x}{2}$$

$$\Rightarrow 4\tan^2\frac{x}{2} - 6\tan\frac{x}{2} - 4 = 0$$

$$\Rightarrow 2\tan^2\frac{x}{2} - 3\tan\frac{x}{2} - 2 = 0$$

The equation is quadratic in $\tan\frac{x}{2}$

$$\Rightarrow \tan\frac{x}{2} = \frac{-(-3)\pm\sqrt{9+16}}{2.2} = \frac{3\pm5}{4} = 2, -\frac{1}{2}$$

Given $\frac{\pi}{2} \le x \le \pi \Rightarrow \frac{\pi}{4} \le \frac{x}{2} \le \frac{\pi}{2} \Rightarrow \frac{x}{2} \in II^{st}$ quadarnt

In II^{st} quadrant, $\tan\frac{x}{2} \ge 0 \Rightarrow \tan\frac{x}{2} = 2$

We know, $1+\tan^2\theta = \sec^2\theta$

$$\Rightarrow 1+\tan^2\frac{x}{2} = \sec^2\frac{x}{2} \Rightarrow 1+(2)^2 = \sec^2\frac{x}{2}$$

$$\Rightarrow \sec^2\frac{x}{2} = 5 \Rightarrow \sec\frac{x}{2} = \pm\sqrt{5} \Rightarrow \cos\frac{x}{2} = \pm\frac{1}{\sqrt{5}}$$

In II^{st} quadrant, $\cos\frac{x}{2} \ge 0 \Rightarrow \cos\frac{x}{2} = \frac{1}{\sqrt{5}}$

We know $\sin\theta = \pm\sqrt{1-\cos^2\theta}$

$$\sin\frac{x}{2} = \pm\sqrt{1-\cos^2\frac{x}{2}} = \pm\sqrt{1-\frac{1}{5}} = \pm\sqrt{\frac{4}{5}} = \pm\frac{2}{\sqrt{5}}$$

In II^{st} quadrant, $\sin\frac{x}{2} \ge 0 \Rightarrow \sin\frac{x}{2} = \frac{2}{\sqrt{5}}$

$\therefore$ (i) $\sin\frac{x}{2} = \frac{2}{\sqrt{5}}$ (ii) $\cos\frac{x}{2} = \frac{1}{\sqrt{5}}$ (iii) $\tan\frac{x}{2} = 2$

27. System of inequations

$x \geq 0, y \geq 0, 5x + 3y \leq 500; x \leq 70$ and $y \leq 125$.

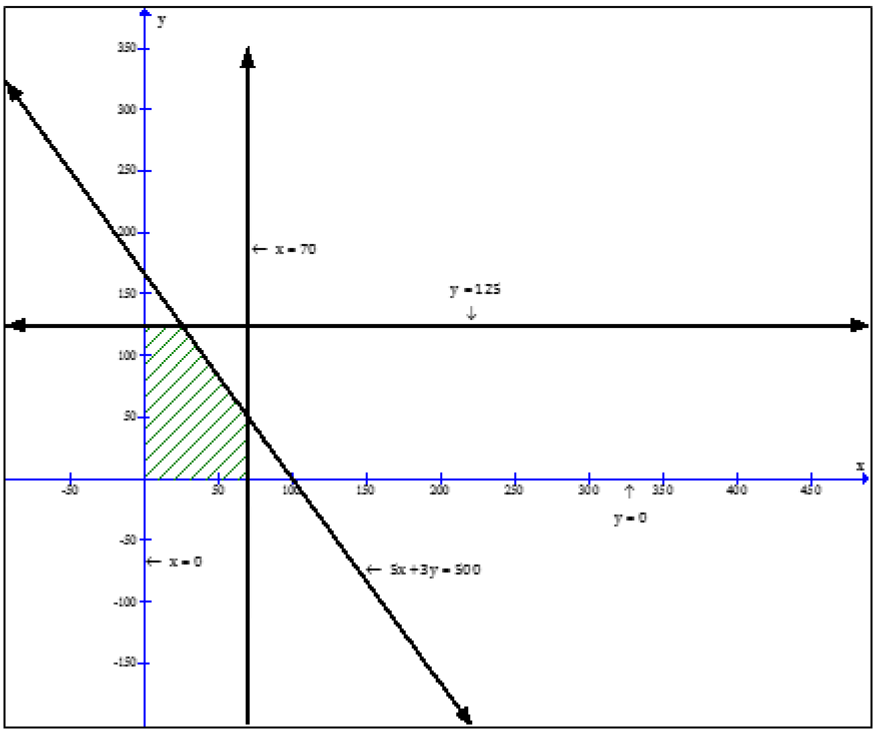
Converting inequations to equations

$5x + 3y = 500 \Rightarrow y = \frac{500-5x}{3}$

x	100	40	-80
y	0	100	300

$x \leq 70$ is $x = 70$ and $y \leq 125$ is $y = 125$.

Plotting these lines and determining the area of each line we get



OR

The amount of acid in 1125 lt of the 45% solution=45% of 1125=$\frac{45 \times 1125}{100}$

Let x lt of the water be added to it to obtain a solution between 25% and 30% solution

$$\Rightarrow 25\% < \frac{1125 \times \frac{45}{100}}{1125 + x} < 30\%$$

$$\Rightarrow \frac{25}{100} < \frac{1125 \times \frac{45}{100}}{1125 + x} < \frac{30}{100}$$

$$\Rightarrow \frac{25}{100} < \frac{1125 \times 45}{(1125 + x) \times 100} < \frac{30}{100}$$

$$\Rightarrow 25 < \frac{1125 \times 45}{(1125 + x)} < 30$$

$$\Rightarrow \frac{1}{25} > \frac{(1125 + x)}{1125 \times 45} > \frac{1}{30}$$

$$\Rightarrow \frac{1125 \times 45}{25} > (1125 + x) > \frac{1125 \times 45}{30}$$

$$\Rightarrow \frac{50625}{25} > (1125 + x) > \frac{50625}{30}$$

$$\Rightarrow 2025 > (1125 + x) > 1687.5$$

$$\Rightarrow 2025 > (1125 + x) > 1687.5$$

$$\Rightarrow 2025 - 1125 > x > 1687.5 - 1125$$

$$\Rightarrow 900 > x > 562.5$$

$$\Rightarrow 562.5 < x < 900$$

So the amount of water to be added must be between 562.5 to 900 lt

28. Let P(n): $5^n - 5$ is divisible by 4 be the given statement.

Let n = 1, 5 - 5 = 0 is divisible by 4

$\Rightarrow$P(1) is true

Let P(k) be true ie $5^k - 5$ is divisible by 4

Let $5^k - 5 = 4m$

To prove the result for n = k + 1 we need to show that $5^{k+1} - 5$ is divisible by 4

$5^{k+1} - 5 = 5^k \times 5 - 5$

$= (4m + 5) \times 5 - 5$

$= 20m + 25 - 5$

$= 4\,(5m + 5)$ is divisible by 4

$\therefore$ Result holds for n = k + 1

$\therefore$ $5^n - 5$ is divisible by 4 for all n.

Let us take another statement

P'(n): $2 \times 7^n + 3 \times 5^n - 5$ is divisible by 24.

For n = 1, $2 \times 7 + 3 \times 5 - 5 = 24$ is divisible by 24

$\therefore$ P'(1) is true

Let P'(k) be true i.e., $2\times7^k + 3\times5^k - 5 = 24q$

To prove: P'(k + 1) is also true

Consider $2\times7^{k+1} + 3\times5^{k+1} - 5 = 2\times7^k\times7 + 3\times5^{k+1} - 5$

$= (24q - 3\times5^k + 5)7 + 3\times5^k\times5 - 5$

$= 24\times7q - 21\times5^k + 35 + 15\times5^k - 5$

$= 24\times7q - 6(5^k - 5)$ (Since $5^k - 5 = 4p$)

$= 24\times7q - 6\times4p$

$= 24(7q - p)$ which is a multiple of 24

$\therefore$ P'(k + 1) is true.

Hence by the principle of Mathematical Induction, the result holds true for all $n\in N$.

29. a, b, and c are in A.P. $\Rightarrow 2b = a + c$

b, c, and d are in G.P. $\Rightarrow c^2 = bd$

$\frac{1}{c}, \frac{1}{d}$, and $\frac{1}{e}$ are in A.P. $\Rightarrow \frac{2}{d} = \frac{1}{c} + \frac{1}{e}$

$d = \frac{2ce}{c+e}$

$c^2 = bd \Rightarrow c^2 = \frac{1}{2}(2b)\,d$

$= \frac{1}{2}(a+c)\left(\frac{2ce}{c+e}\right)$

$\Rightarrow (c+e)\,c^2 = ce\,(a+c)$

$\Rightarrow (c+e)\,c = e\,(a+c)$

$\Rightarrow c^2 + ec = ea + ec$

$\Rightarrow c^2 = ea$

$\Rightarrow$ a, c, and e are in G.P.

OR

$$\frac{1\times2^2 + 2\times3^2 + ...+ n\times(n+1)^2}{1^2\times2 + 2^2\times3 + ...+ n^2\times(n+1)}$$

$$= \frac{\sum_{k=1}^{n} k(k+1)^2}{\sum_{k=1}^{n} K^2(k+1)}$$

$$= \frac{\sum k^3 + 2\sum k^2 + \sum k}{\sum k^3 + \sum K^2}$$

$$= \frac{\left[\frac{n(n+1)}{2}\right]^2 + 2\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2 + \frac{n(n+1)(2n+1)}{6}}$$

$$=\frac{\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2}+\frac{2(2n+1)}{3}+1\right]}{\frac{(n(n+1)}{2}\left[\frac{n(n+1)}{2}+\frac{2n+1}{3}\right]}$$

$$=\frac{3n^2+3n+8n+4+6}{3n^2+3n+4n+2}$$

$$=\frac{3n^2+11n+10}{3n^2+7n+2}$$

$$\frac{(n+2)(3n+5)}{(n+2)(3n+1)}=\frac{3n+5}{3n+1}$$